




Dealing with difficulties in maths by using authentic materials



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Dealing with difficulties in maths by using authentic materials

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MLD effective interventions

TABLE 1
Simple comparisons of all effects (g) and heterogeneity (Q).

| Instructional component | Random effects mean | Median |
|---|---------------------|--------|
| Explicit instruction ($n = 11$) | 1.22**** | 1.39 |
| Use of heuristics ($n = 4$) | 1.56**** | 1.62 |
| Student verbalizations ($n = 8$) | 1.04**** | 1.42 |
| Visuals for teacher and student ($n = 7$) | 0.46**** | 0.67 |
| Visuals for teacher only ($n = 5$) | 0.41** | 0.50 |
| Visuals combined ($n = 12$) | 0.47**** | 0.52 |
| Sequence and/or range ($n = 9$) | 0.82**** | 0.54 |
| Teacher feedback ($n = 7$) | 0.21** | 0.19 |
| Teacher feedback plus options ($n = 3$) | 0.34* | 0.40 |
| Teacher feedback combined ($n = 10$) | 0.23**** | 0.21 |
| Student feedback ($n = 7$) | 0.23**** | 0.17 |
| Student feedback with goal setting ($n = 5$) | 0.17 | -0.17 |
| Student feedback combined ($n = 12$) | 0.21** | 0.14 |
| Cross-age tutoring* ($n = 2$) | 1.02**** | 0.95 |
| Peer-assisted learning within a class ($n = 6$) | 0.14 | 0.17 |

Note. n refers to number of effects.
a. Fewer observed effects ($n = 2$) reduces confidence cross-grade c
* $p < .10$. ** $p < .05$. *** $p < .01$. **** $p < .001$.

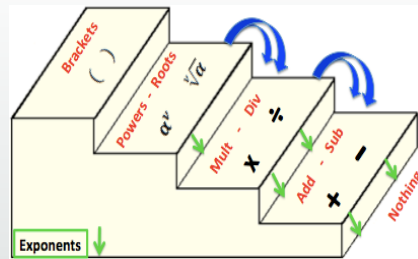
(Gersten et al., 2009)

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Use of heuristics

- A heuristic is a method or strategy that **exemplifies a generic approach** for computational skills, problem solving, solving an equation, etc.
- Instruction in heuristics, unlike direct instruction, **is not problem-specific**.
- Heuristics can be used in **organizing information** and solving a range of math problems.

$$2 \cdot (4 \cdot 6 - 3 \cdot 5) + 24 \div (3 + 3^2) - 4 \cdot 2^2 + 1^3 =$$



$$\frac{2(x-1)}{3} - 1 = x - \frac{5-3x}{4}$$

$$(x-3)^2 - x(x-6) - 8 =$$

(Yayanthi et al., 2008)

C
O
S
M
O
S

Problem:
Maria started reading an 120 page book on Monday.
If she read 38 pages on Monday and 25 pages on
Tuesday, how many pages are left to finish the book?

?

| | | | |
|----|-----|---|-------|
| | 120 | | |
| 38 | 25 | ? | = 120 |

Estimation: Operations:

| | |
|--|---|
| $\begin{array}{r} 38 \\ + 25 \\ \hline 63 \end{array}$ | $\begin{array}{r} 120 \\ - 63 \\ \hline 57 \end{array}$ |
|--|---|

Validation:

Answer:

Circle

Organize

Sketch

Mind guess

Operate

Scan

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Explicit instruction

- Clear modeling of the solution **specific to the problem**.
- **Thinking** the specific steps **aloud** during modeling,
- Presenting **multiple examples** of the problem.
- Providing immediate **corrective feedback** to the students on their accuracy.

$$47 - 23 = \quad \textcircled{20} \quad \textcircled{+4} \quad \longrightarrow \quad 24$$

$$64 - 25 = \quad \textcircled{40} \quad \textcircled{-1} \quad \longrightarrow \quad 39$$

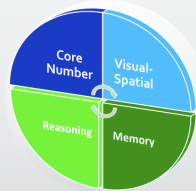
$$83 - 36 = \quad \textcircled{50} \quad \textcircled{-3} \quad \longrightarrow \quad 47$$

Clever circles provide clear visual image, language and symbols of the addends **avoiding the memory overload** of traditional methods and **not obscuring the meaning of the digits**.

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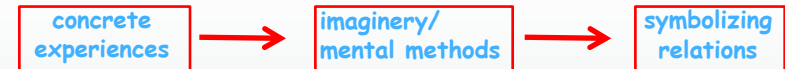
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- ❑ Research has shown that an important stage between the actual manipulation of objects and abstract work with numerical symbols is a stage in which **objects are imagined** (Hughes, 1986).
- ❑ **Real life** activities and scenarios as well as **authentic materials** using will motivate children to find answers to numerical problems (Beisheuizen, 1995).

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Hundreds - Tens - Units

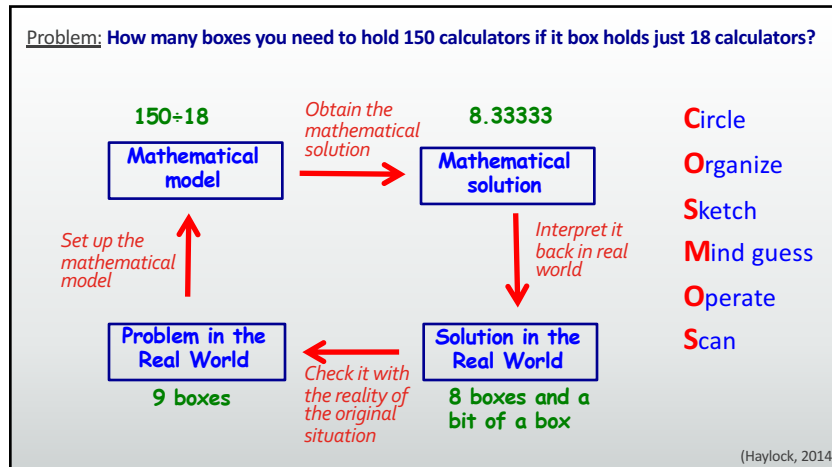
| 100 | 10 | 1 |
|-----|----|---|
| E | Δ | M |
| 2 | 3 | 6 |

Number-cards enable children to make helpful connections between the visual image of the cards, language and symbols (Faux, 1998).

CONCEPTUAL INTERVENTION

- Teaching approaches focus on the links that demonstrate the **logical structure** underlying numbers and number operations.
- Math information is most likely to get stored if it **makes sense** and **has meaning**.
- Students who see mathematics as rules or procedures to be memorized are not only **prone to struggle** in higher grade level but also are likely to develop **negative attitudes** about the subject (Richland et al., 2012).
- Students can grasp high-level ideas but **they will not develop the brain connections** that allow them to do so if **they are given low-level work** and negative messages about their own potential (Boaler & Foster, 2014).

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① Greg had 225€ in his saving account. He spent 20% of his money on a present for his friend. How much money does Greg have left?

100% → 225
1% → $\frac{225}{100}$
80% → $\frac{225}{100} \cdot 80 = 180$

225
100%

80% 20%

② Scott lunched with his friend at the Happy Hamburger. The total bill was 20€. They decided to leave a 15% tip for the waiter. How much was the lunch cost?

100% → 20
1% → $\frac{20}{100}$
115% → $\frac{20}{100} \cdot 115 = 23$

20
100%

100% 15%

C
O
S
M
O
S

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Stimulating number sense

Activity 1: Give me a card

Activity 2: Cards' name

Activity 3: From 1 to 10

Activity 4: More or less than 5?

$$x+y=?$$

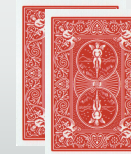
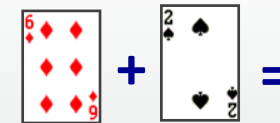
$$x+x^2=?$$

$$x+x=?$$

$$6+2=?$$

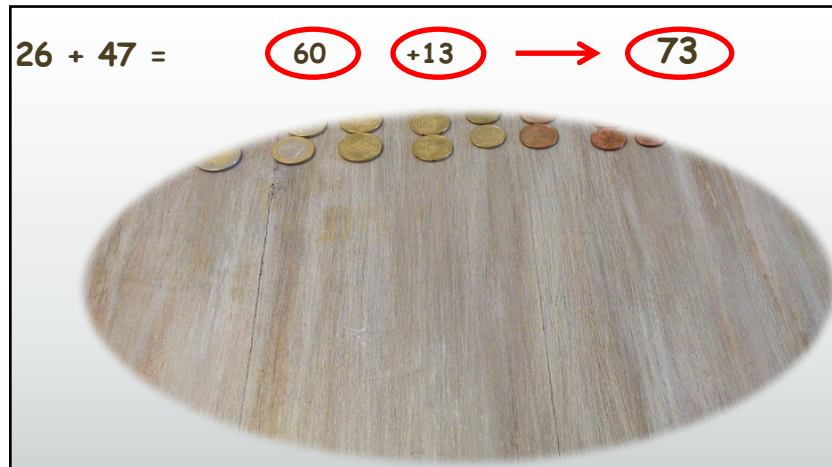
$$2+6=?$$

Addition concept



Card +1

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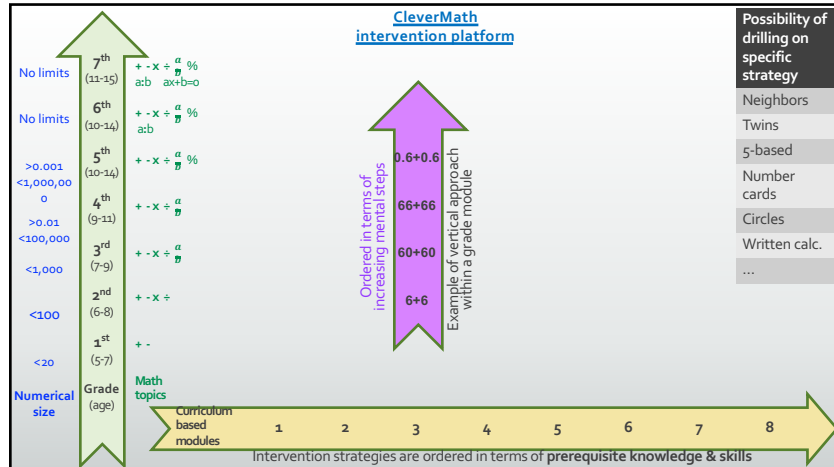
[CleverMath intervention](#)

An intelligent tutor system to deal with difficulties in learning mathematics

- Cognitive psychology, artificial intelligence, and computer technology have advanced to the point where it is feasible to build computer **Intelligent Tutoring Systems** (ITSs) that are effective and intelligent.
- Computer-assisted instruction software in the past tended to take a simplistic approach to children's errors and to reward correct answers, and reject incorrect answers, without scope for analyzing how the errors occurred (Hativa, 1988).
- ITSs are being developed to provide the student with nearly the same instructional advantage that a sophisticated **human tutor can provide**. A good private tutor understands the student and responds to the student's special needs.
- CleverMath platform provides **informative feedback**: a) interpretation of the mistake b) compensatory materials or exercises in different representation c) the correct answer

Example: circles

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| | 1.000 | 100 | 10 |
|--------------------|-------|-----|----|
| Decimal navigation | 9 0 0 | 9 0 | 9 |
| | 8 0 0 | 8 0 | 8 |
| | 7 0 0 | 7 0 | 7 |
| | 6 0 0 | 6 0 | 6 |
| | 5 0 0 | 5 0 | 5 |
| Number cards | 4 0 0 | 4 0 | 4 |
| | 3 0 0 | 3 0 | 3 |
| | 2 0 0 | 2 0 | 2 |
| | 1 0 0 | 1 0 | 1 |
| | 0 | 0 | 0 |

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Vertical additions with digits

5 of clubs + 2 of spades = 7 of diamonds + 3 of spades + 6 of diamonds = 9 of hearts

6 of diamonds + 2 of spades = 8 of hearts

7 of spades + 5 of clubs = 12 (2 of spades + 10 of hearts)

Subtraction concept

$3x - x = ?$
 $x^2 - x = ?$

$6 - 2 = ?$
 $2 - 6 = ?$

6 of diamonds - 2 of spades =

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Subtract by partitioning to units (smart circles)

$$47 - 23 = \overset{10}{\textcircled{20}} \overset{1}{\textcircled{+4}} \longrightarrow \textcircled{24}$$

$$64 - 25 = \textcircled{40} \textcircled{-1} \longrightarrow \textcircled{39}$$

$$86 - 49 = \textcircled{40} \textcircled{-3} \longrightarrow \textcircled{37}$$

$$346 - 125 = \textcircled{200} \textcircled{+20} \textcircled{+1} \longrightarrow \textcircled{221}$$



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Vertical subtractions with digits

7563 - 1234

5235 - 3741

$x \cdot y = ?$ Multiplication concept

3x2 = ?


2x3 = ?











3 of spades \times 2 of hearts =

Repeated addition

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Time tables




| | | | | |
|--|--|--|--|--|
|  4 |  8 |  12 |  16 |  20 |
|  24 |  28 |  32 |  36 |  40 |

Division concept


$6 \div 2 = ?$
 $2 \div 6 = ?$



Grouping



Sharing



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Visuals

Add integer numbers

$(+5) + (+2) = +7$

$(+5) + (-2) = +3$

$(-5) + (-2) = -7$

$(-5) + (+2) = -3$

Core
Number

Visual-
Spatial

Reasoning

Memory

Effective instruction involves an interplay of concepts and procedures (Barody et al., 2007; Osana et al., 2013; Richland et al., 2012).

Addition

$(+5) + (+2) = +7$

$(+5) + (-2) = +3$

$(-5) + (-2) = -7$

$(-5) + (+2) = -3$

Subtraction

$(+5) - (-2) =$

$(+5) - (+2) = +3$

$(-5) - (+2) =$

$(-5) - (-2) = -3$

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$(+5) - (-3) + (-2) - (+4) =$ $(-4) + (+2) - (-8) - (+5) - (+9) =$
 $(+5) + (+3) + (-2) + (-4) = +2$ $(-4) + (+2) + (+8) + (-5) + (-9) = -8$

$-4 + 2 + 8 - 5 - 9 = -8$

CRA in algebra

1 x x^2 x^3

From research to practice

$x^2 + x = x^3$

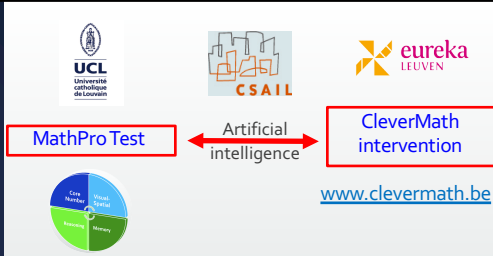
$3x + 4 + 2x + 3 = 5x + 7$

$3x^2 - x^2 = 2x^2$

Concrete – Representational-Abstract (CRA) is an excellent example of linking conceptual and procedural understanding through manipulatives, drawings or pictorial representations (Butler et al., 2003; Gersten et al., 2009; Witzel, 2016).

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Future directions



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CSAIL

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MathPro Test

Artificial intelligence

CleverMath intervention

www.clevermath.be

- Test the efficacy of the CleverMath intervention platform
 - ✓ Use of the CleverMath platform (open-trial design)
 - ✓ *The importance of adapting the system to the child's errors*



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Thanks!

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